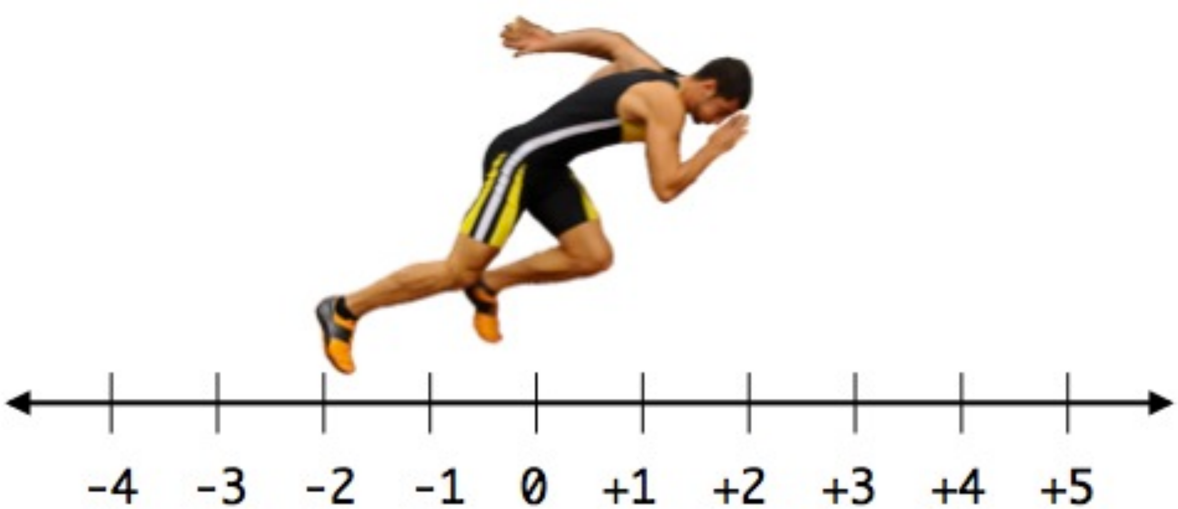
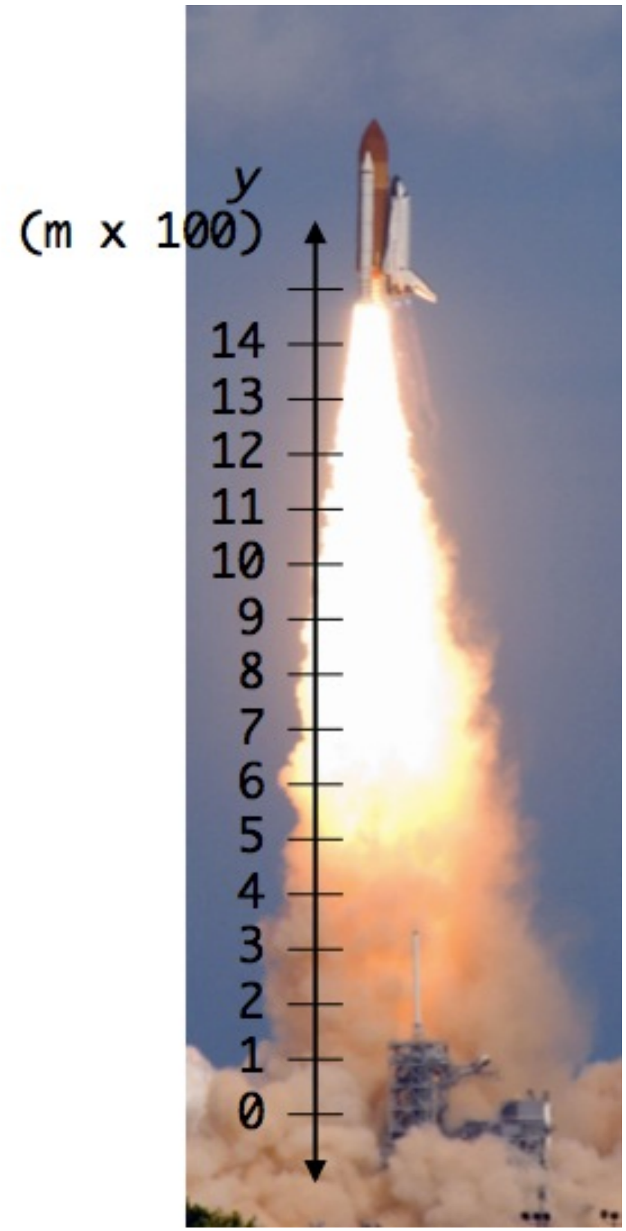
Motion in Two Dimensions

Motion along an axis

Up this point, we've been looking at linear motion that happens along the *x*-axis or the *y*-axis, with the direction of a vector indicated by a + or – sign.





Adding vectors

What happens when two motions are combined? How can we analyze the resulting (*resultant*) motion?

What is the *net* velocity of a jet flying through the air at 1000 mi/hr, straight into a 100 mi/hr headwind?

Graphically adding vectors

You can do this problem in your head, but we need to be able to use a system for solving more difficult problems, so let's draw a picture.

What is the *net* velocity of a jet flying through the air at 1000 mi/hr, straight into a 100 mi/hr headwind?

Resultant velocity = 1000 mi/hr east + 100 mi/hr west
Resultant velocity = 1000 mi/hr – 100 mi/hr
Resultant velocity = 900 mi/hr east

Example

Consider a confused teacher who walks 5 meters to the *left* in the room, and then 3 meters to the *right*.

a. Draw a diagram of the teacher' s motion.
b. Calculate the teacher's *distance* traveled.
c. Calculate the teacher's *displacement*.
d. If the teacher did all of this in 2.0 seconds, calculate her *speed*.
e. If the teacher did all of this in 2.0 seconds, calculate her *velocity*.

b. 5+3=8m

c. -5(left) +3(right)=-2m

d.
$$s = \frac{d}{t}$$
$$s = \frac{8m}{2s} = 4m/s$$

e.
$$v = \frac{\Delta x}{t}$$
$$v = \frac{-2m}{2s} = -1m/s$$

Graphically adding 2-D vectors

Now let's apply the same process to a two-dimensional problem.

What is the *net* velocity of a jet flying through the air at 400 mi/hr east while a (really strong!) wind is blowing 300 mi/hr to the north?

$$v_{resultant} = v_x + v_y$$

$$v_{resultant} = \sqrt{v_x^2 + v_y^2}$$

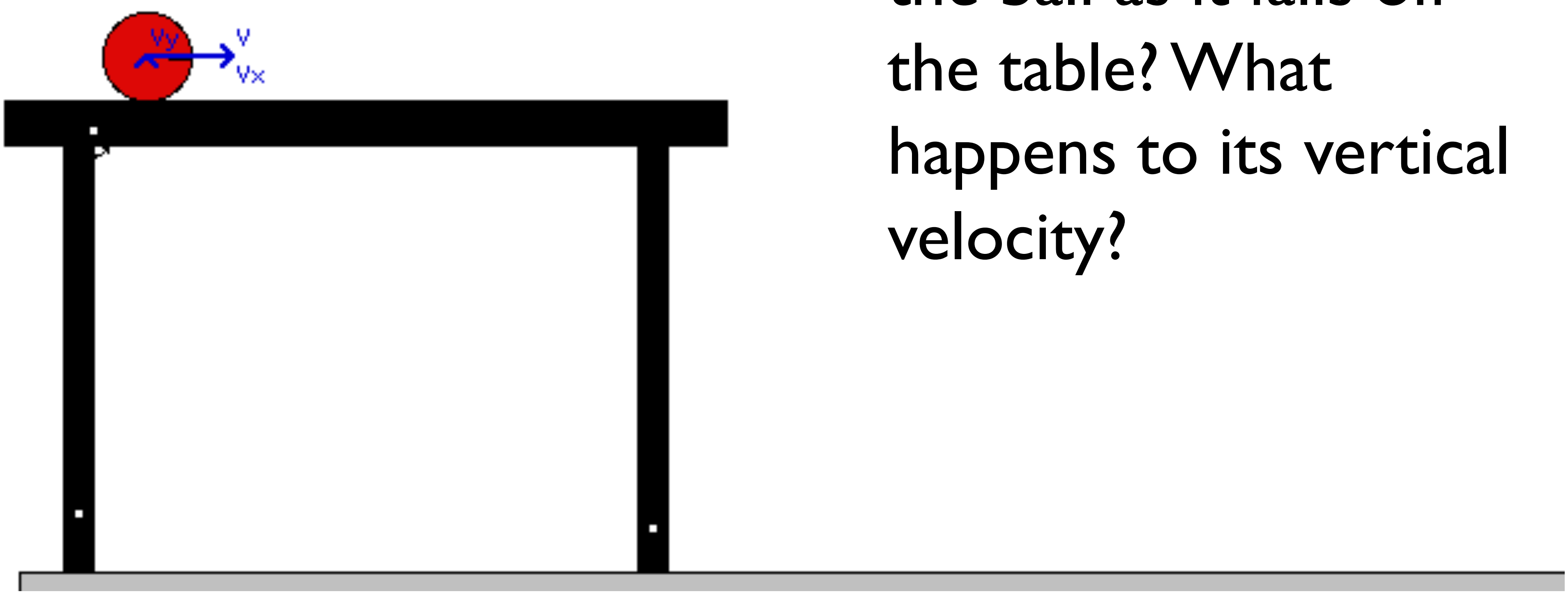
$$v_{resultant} = \sqrt{400^2 + 300^2}$$

$$v_{resultant} = 500 mi/hr, northeast$$

Intro to Projectile Motion

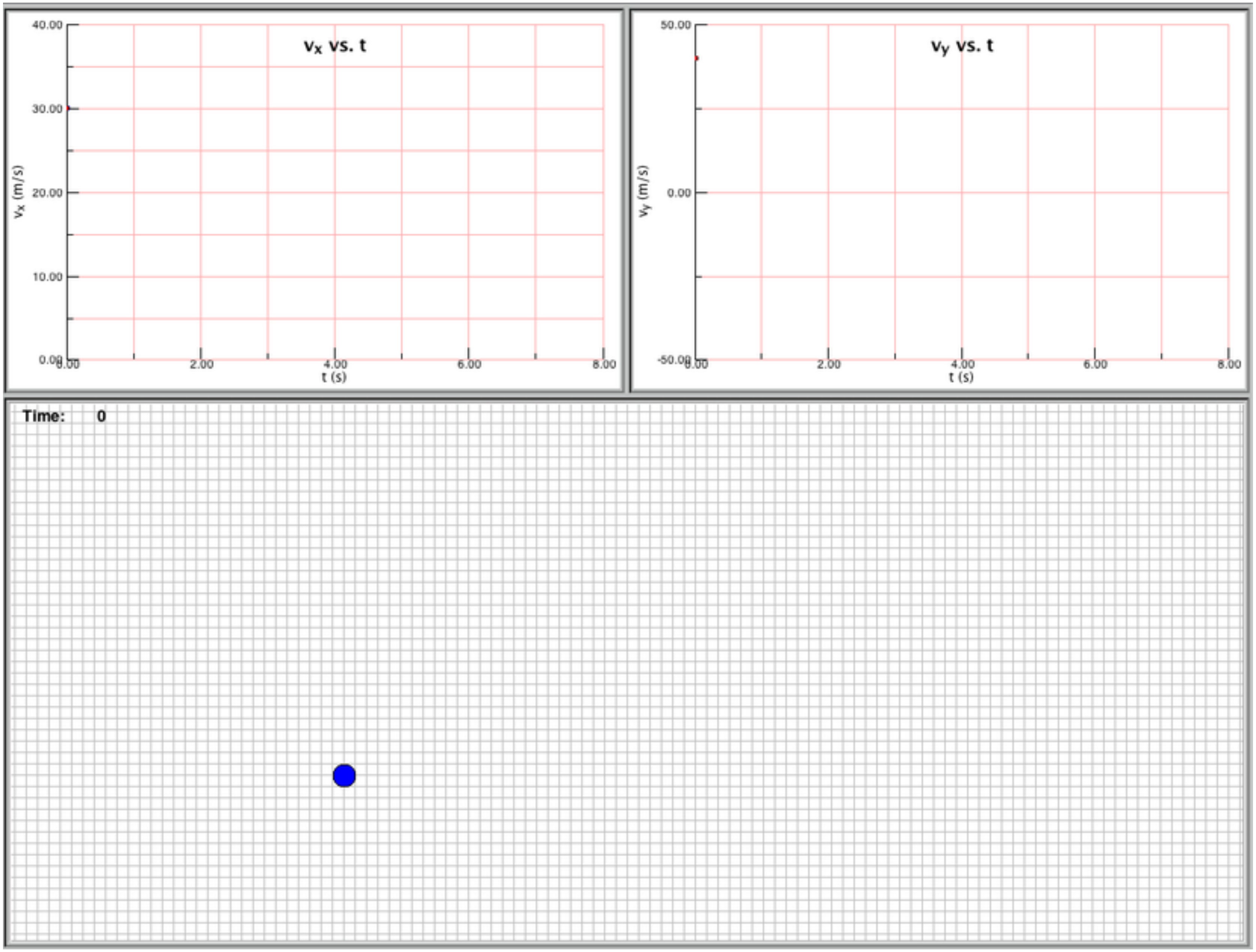
This process of looking at *x* and *y* motions separately, but combining them together to see what's happening in the big picture, is extremely useful, and something that we'll be doing over and over again as we examine *projectile motion*–the motion of objects moving through the air.

Graphics of Projectile Motion

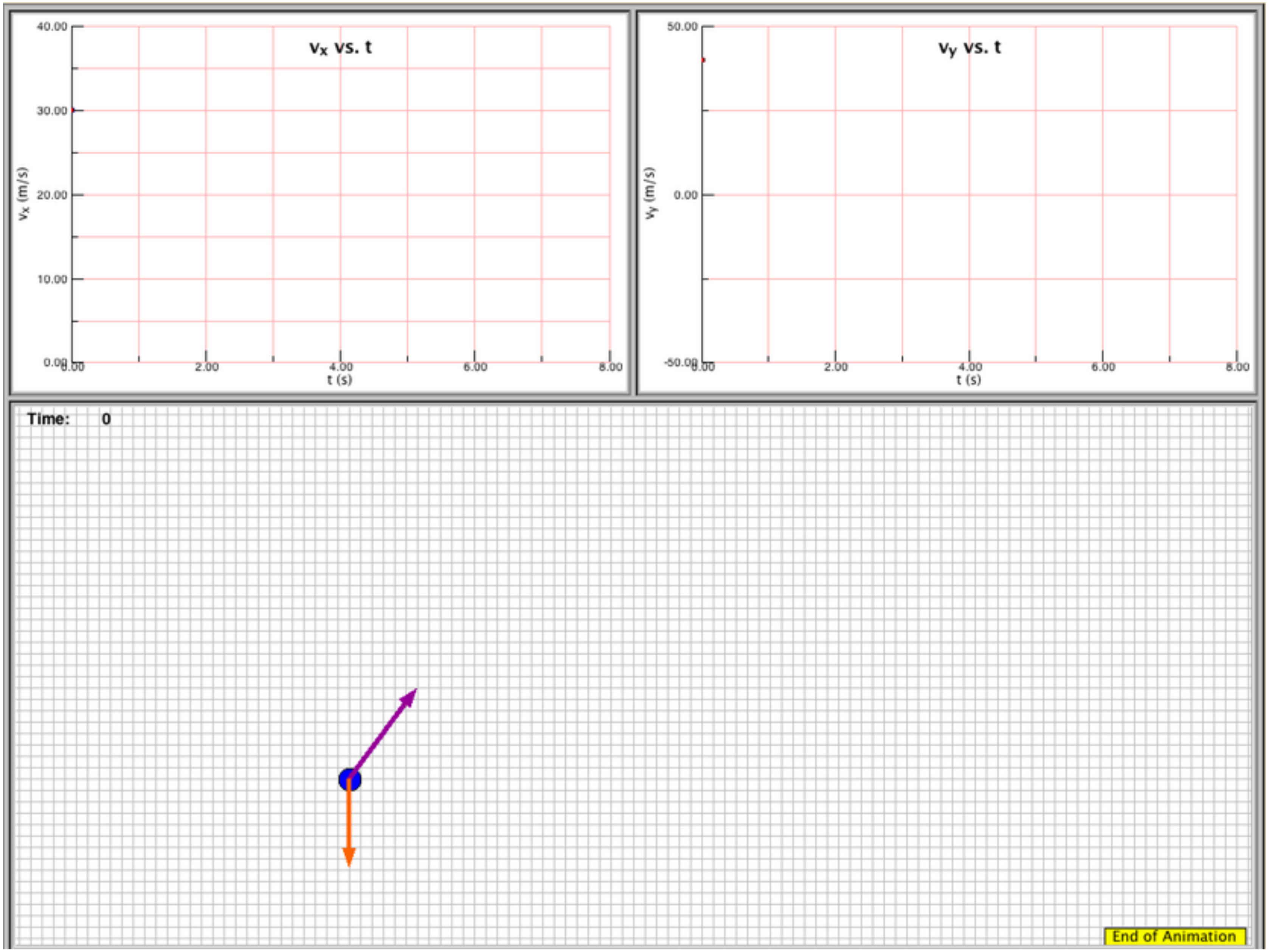


What happens to the horizontal velocity of the ball as it falls off the table? What happens to its vertical velocity?

Graphics of Projectile Motion



Graphics of Projectile Motion



In Projectile Motion...

The horizontal velocity of the object ______________, because ____.

The vertical velocity of the object ________________, because ____.

Example

A ball is rolling at 2.0 m/s along a table when it rolls off the edge, which is 1.0 m above the ground.

a. Draw a sketch of the problem.
b. How fast is the ball moving horizontally after it leaves the table?
c. How much time passes before the ball hits the ground?
d. How far away from the edge of the table does the ball land?

b. 2.0 m/s

c. $d = v_i t + \frac{1}{2}at^2$

$$t_{fall} = \sqrt{\frac{2d}{g}} = 0.14s$$

d. $x = vt$

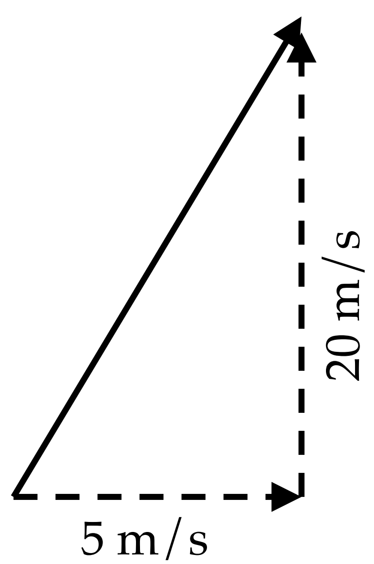
$$x = (2.0m/s)(0.14s)$$

$$x = 0.28m$$

$$y = -1.0m$$

Example

Consider a frog, jumping with an initial vertical velocity of 20 m/s and an initial horizontal velocity of 5 m/s. How high does the frog jump? How far away does he land?



Vertical analysis:

$$v_y = 20m/s$$

$$v_f = v_i + at$$

$$0 = 20m/s + (-9.8)t$$

$$t = 2.04s$$

$$\Delta y = \frac{1}{2}gt^2$$

$$For\ falling: \Delta y = \frac{1}{2}(-9.8)(2.04)^2$$

$$\Delta y = 20.4m$$

Horizontal analysis:

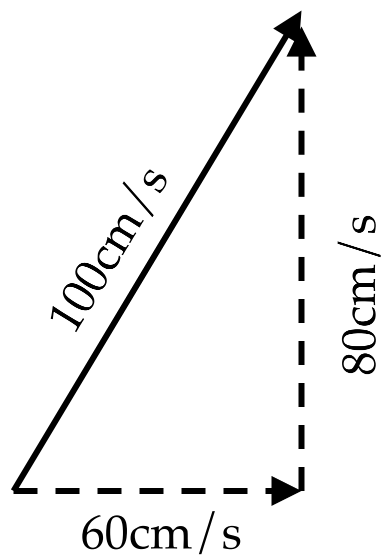
$$v_x = 5m/s$$

$$\Delta x = vt$$

$$\Delta x = (5m/s)(2.04s) = 10.2m$$

Example

Consider a frog, jumping with an initial velocity of 100 cm/s at 53° above horizontal. How high does the frog jump? How far away does he land?



$$\textit{Vertical analysis}:$$

$$v_y = 80cm/s$$

$$v_f = v_i + at$$

$$0 = 0.8m/s + (-9.8)t$$

$$t = 0.082s$$

$$\Delta y = \frac{1}{2}gt^2$$

$$\textit{For falling}: \Delta y = \frac{1}{2}(-9.8)(0.082)^2$$

$$\Delta y = 0.032m$$

$$\textit{Horizontal analysis}:$$

$$v_x = 60cm/s$$

$$\Delta x = vt$$

$$\Delta x = (0.60m/s)(0.082s) = 0.049m$$