Conservation of Momentum

Newton: Quantity of Motion

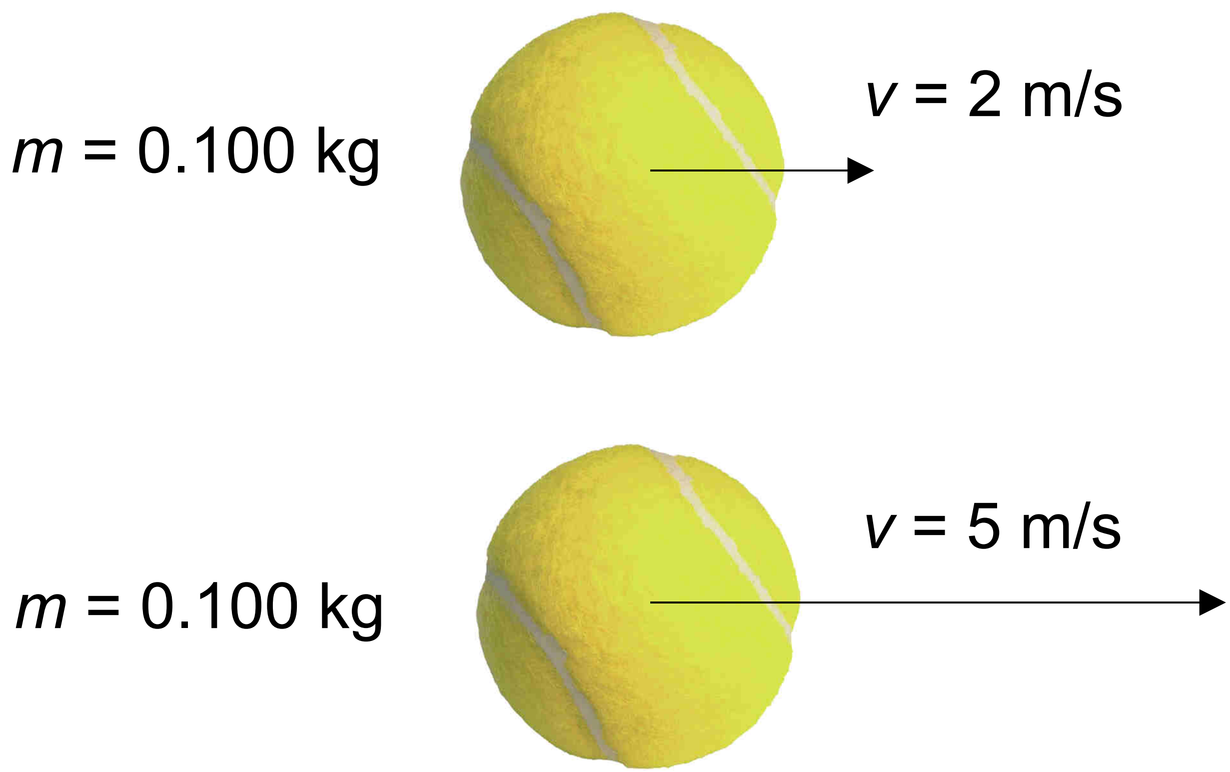
Newton, in describing moving objects, talked about their "quantity of motion," a value based both on the inertia (*mass*) of the object and its *velocity*.

"Quantity of motion" is *momentum*.

$$\textit{Momentum} \quad \vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

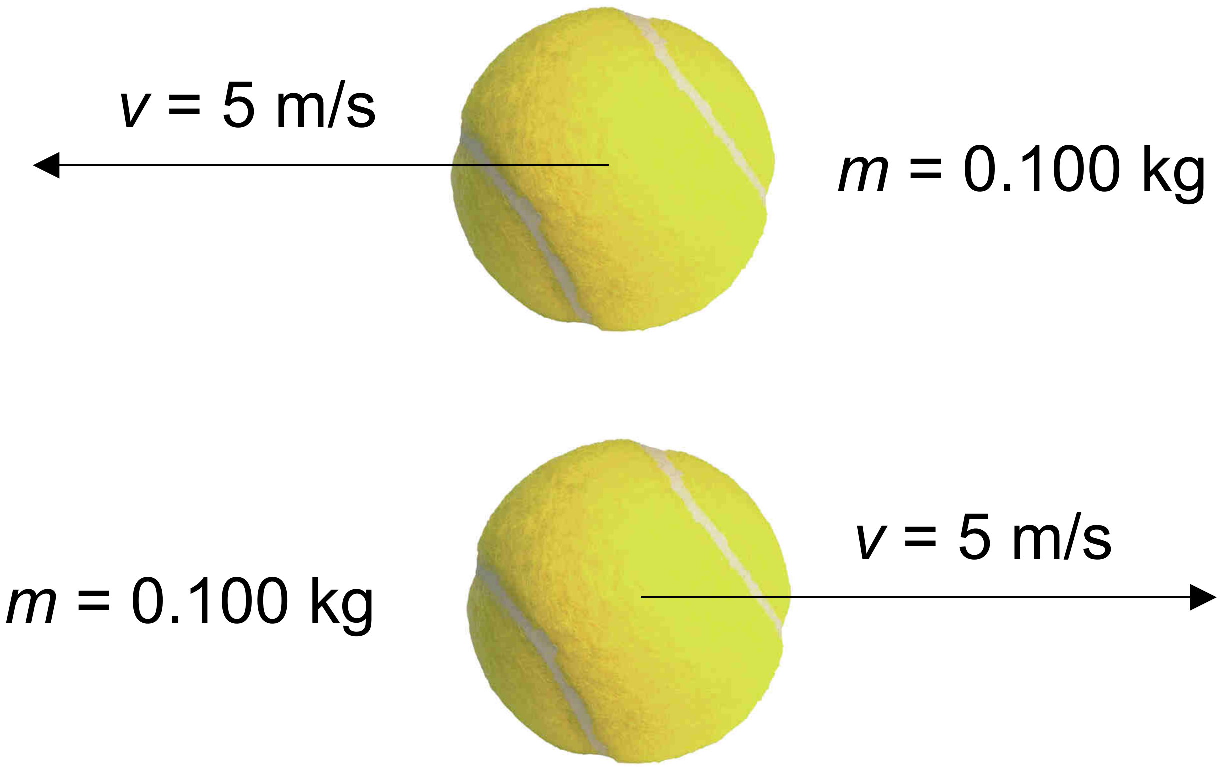
Comparing momenta

Which of these two objects has more momentum?



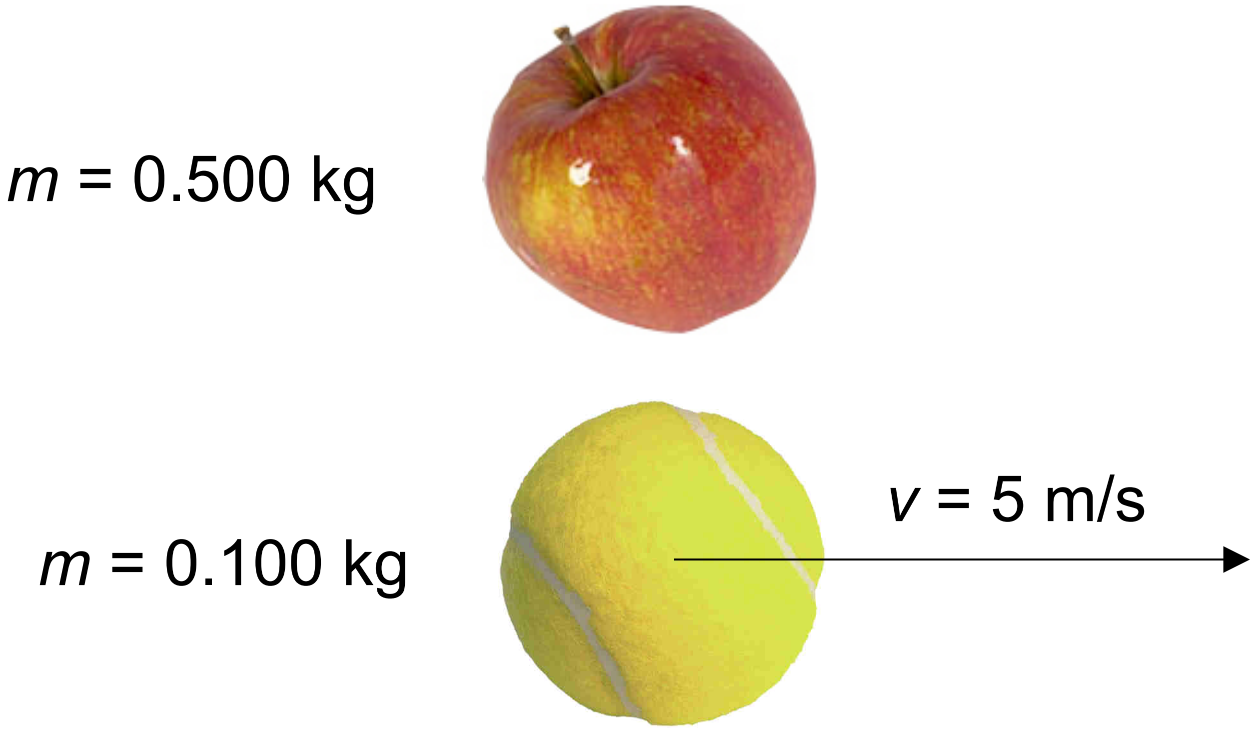
Comparing momenta

Do these two objects have the same momentum?



Comparing momenta

What would you have to do to give the apple the same momentum as the tennis ball?



Details

- An object with no velocity has momentum = _______
- An object with negative velocity has ___________ momentum.
- The units for momentum are _________

Momentum and Force

We've learned Newton's 2nd Law as F_{net}=ma… but that's not how he originally thought about it.

Newton stated that a Force acting over a time causes a change in an object's "quantity of motion."

$$\mathbf{F}_{net} t = \Delta \mathbf{p}$$

$$\mathbf{F}_{net} = m\mathbf{a}$$

$$\mathbf{F}_{net} = m\frac{\mathbf{v}_f - \mathbf{v}_i}{\Delta t}$$

$$\mathbf{F}_{net} = \frac{m\mathbf{v}_f - m\mathbf{v}_i}{\Delta t}$$

$$\mathbf{F}_{net} = \frac{\Delta \mathbf{p}}{\Delta t}$$

Momentum and Impulse

An *impulse* produces a *change in momentum*.

$$\mathbf{F}_{net}t = \Delta\,\mathbf{p}$$

"Impulse"

"change in momentum"

Example 1

A 50-gram tennis ball is traveling at 25 m/s when it's hit by a racket.

0.1s later, the ball is traveling at 30 m/s in the opposite direction. How much force did the racket apply to the ball?

$$F\Delta t = m\Delta v$$

$$F = m\frac{\Delta v}{\Delta t} = m\frac{v_{final} - v_{initial}}{t}$$

$$F = (0.050kg)\frac{(-30m/s)-(+25m/s)}{0.1s}$$

$$F = -27.5N$$

Impulse = Force x time

Why do catchers wear such fat mitts?

$$F\,t = m\Delta v \qquad\qquad F\,t = m\Delta v$$

Law of Conservation of Momentum

Whenever two isolated, unchanged particles interact with each other, their total momentum remains constant.

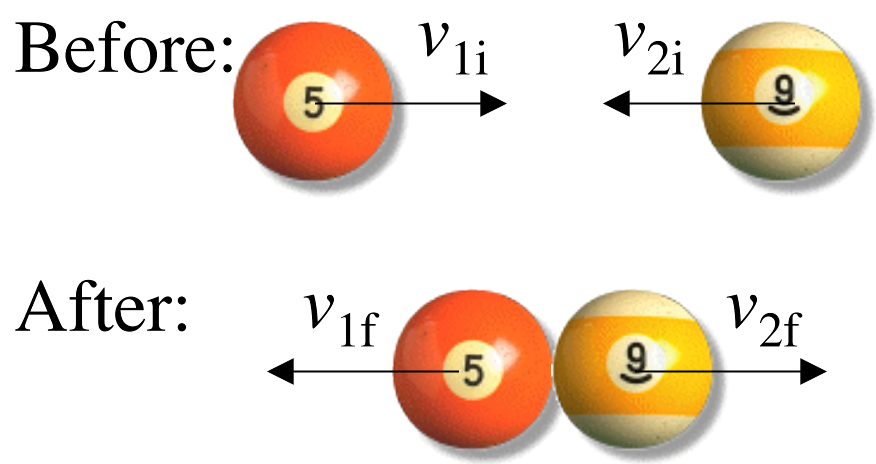
$$p_1 + p_2 = p_1' + p_2'$$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v'$$

Collision

What happens when two billiard balls collide?



$$F = \frac{\Delta p}{\Delta t} \text{(Newton's 2nd Law)}$$

$$F\Delta t = \Delta p = m_f v_f - m_i v_i$$

$$F_{5,9}\Delta t = \Delta p = m_{5f} v_{5f} - m_{5i} v_{5i}$$

$$F_{5,9} = -F_{9,5} \text{ so we can write :}$$

$$m_{5f} v_{5f} - m_{5i} v_{5i} = -(m_{9f} v_{9f} - m_{9i} v_{9i})$$

Rearrange to get :

$$m_{9i} v_{9i} + m_{5i} v_{5i} = m_{9f} v_{9f} + m_{5f} v_{5f}$$

Example

Alex (m=75 kg) sits on a 5kg cart with no-friction wheels, and gets hit by a 7.0 kg bowling ball with a velocity of 5.0 m/s.

a) What is Alex's velocity after catching the ball?

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_{final}$$

$$(7.0kg)(5.0m/s) + (75kg)(0) = (7 + 75) v_{final}$$

$$v_{final} = \frac{35kg \bullet m/s}{82kg} = 0.43m/s$$

b) What is Alex's velocity after the collision if the bowling ball bounces off with a velocity of -0.50 m/s?

$$m_1 v_1 + m_2 v_2 = m_1 v_{1-final} + m_2 v_{2-final}$$

$$(7.0kg)(5.0m/s) + (75kg)(0) = (7)(-0.5) + (75) v_{final}$$

$$v_{final} = \frac{35 + 3.5}{75} = 0.51m/s$$

Example

A 10,000 kg train car traveling at 24.0 m/s hits an 20,000 kg series of cars at rest. If the cars lock together as a result of the collision, what is their speed afterwards?

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A 10,000 kg train car traveling at 24.0 m/s hits an 20,000 kg series of cars at rest. If the cars at rest are traveling at 16 m/s AFTER the collision, what is the final velocity of the 10,000 kg train car?

Example

A 4.00 kg gun with a 80 cm long barrel fires a 50-gram bullet with at a velocity of 400 m/s. Find:

a) recoil velocity of gun
b) impulse on bullet
c) average acceleration of the bullet
d) time the bullet accelerated
e) force applied to the gun by the bullet

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$$a)\ m_1 v_1 + m_2 v_2 = m_1 v_{1final} + m_2 v_{2final}$$
$$0 = (4kg)v_{1final} + (0.050kg)(400m/s)$$
$$v_{1final} = -5.0m/s$$

$$b)\ \text{Impulse} = Ft = m\Delta v$$
$$= (0.050kg)(400-0) = 20kg \bullet m/s$$

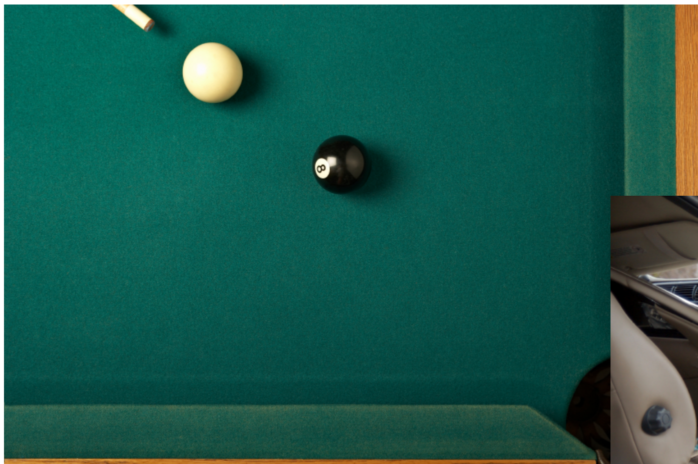
$$c)\ t = \frac{distance}{speed} = \frac{0.80m}{200m/s} = 0.004s$$

$$d)\ a = \frac{v_f - v_i}{t} = \frac{400m/s - 0}{0.004s} = 1.0e4m/s^2$$

$$e)\ F = ma = (0.050kg)(1.0e4m/s^2)$$
$$F = 500N$$

Collisions

Elastic



Inelastic

Perfectly inelastic

Collisions

Momentum is always conserved

$$\sum \Delta p = 0, \text{ or } p_1 + p_2 = p_1' + p_2'$$

Energy is always conserved

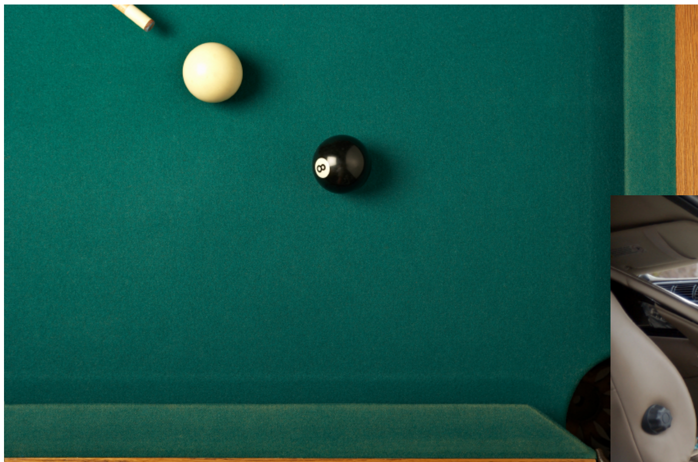
$$\sum \Delta E = 0, \text{ or } \sum E_i = \sum E_f$$

In *some* collisions, there is very little energy "lost" to heat (sound, deformation). In these *elastic collisions*, kinetic energy is conserved:

$$K_1 + K_2 = K_1' + K_2'$$

Collisions

Elastic



Inelastic

Perfectly inelastic

Collisions

- *Elastic collisions*: $K_1+K_2=K_1'+K_2'$ & $\mathbf{p}_1+\mathbf{p}_2=\mathbf{p}_1'+\mathbf{p}_2'$

$$\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2=\frac{1}{2}m_1v_1'^2+\frac{1}{2}m_2v_2'^2 \quad \&$$

$$m_1v_1+m_2v_2=m_1v_1'+m_2v_2'$$

- *Inelastic collisions:*

$$\mathbf{p}_1+\mathbf{p}_2=\mathbf{p}_1'+\mathbf{p}_2' \quad \text{or}$$

$$m_1\mathbf{v}_1+m_2\mathbf{v}_2=m_1\mathbf{v}_1'+m_2\mathbf{v}_2'$$

- ***Perfectly** inelastic collisions:*

$$\mathbf{p}_1+\mathbf{p}_2=\mathbf{p}_1'+\mathbf{p}_2' \quad \text{or}$$

$$m_1\mathbf{v}_1+m_2\mathbf{v}_2=(m_1+m_2)\mathbf{v}_2'$$